FURTHER APPROXIMATIONS TO THE SUN'S DISTANCE.

By Pliny Earle Chase.

(*Read before the American Philosophical Society, April 5th,* 1872.)

If it be true, as is commonly and very plausibly supposed, that molecular and cosmical laws have many significant analogies which are yet undiscovered, it may be well to seek for such analogies wherever we may reasonably hope to find them.

The height of oscillation which I have assumed as the measure of the igneous energy of combustibles, is less, and the resulting estimate of solar mass and distance is greater, if the combustible is dense, composite, or of small specific heat, than if it is rare, simple, or of great specific heat. It seems likely that even hydrogen, the most volatile of all known substances, may have undergone some condensation and loss of specific heat, and that, therefore, my first estimates by flame analysis* were all slightly in excess. This opinion is the more probable, from the fact that the mean of the most recent astronomical estimates of solar distance, is nearly one per cent. less than the mean of the flame estimates.

In searching for some clue to the coëfficient of condensation in hydrogen, if we accept the hypothesis that the luminiferous æther is a perfectly elastic material medium, we may, perhaps, be able to detect some important relations between the velocity of luminous or thermal undulations, and the velocity of oscillations which are directly traceable to gravitating action. In the primary radiation and subsequent double concentration of exploding hydrogen, there is not only a joint attraction of the gaseous particles for each other and of the whole for the earth, but there is also a generation of luminous vibrations, with a velocity such as would be produced by a gravitating force $g = \frac{v^2}{h}$. Equivalent velocities may be generated by masses of different magnitudes, provided the motion is orbital with radii varying as the masses, or the fall is virtually continued to the centre from heights equivalent to twice those proportionate radii. With these preliminary considerations I invite attention to the following coincidences :

1. At the centre of oscillation of the extreme excursion of exploding H_2O before its fall towards the centre of condensation, ($\frac{2}{3}$ of 1009.877 miles from the commencement of the fall, or $\frac{1}{3} \times 1009.877 = 336.626$ miles above the earth's surface,) the velocity imparted by terrestrial gravity in one year $\left(\frac{32.0894377 \times 31558150}{5280} \div \left(\frac{3962.818 + 336.626}{3962.818}\right)^{\frac{1}{2}} = 184{,}130 \text{ miles}\right)$ would be closely coincident with the velocity of light. If the coincidence is exact, $\gamma_{\!/}$ (the Sun's distance) is $497.827 \times 184{,}130 = 91{,}665{,}370$ miles. The value of h corresponding to this distance $\frac{2gt^2r^2}{4\pi^2\gamma_{\!/}^2} = 573.099$ miles, which is 1.0215 times the experimental h (561.043 m).

* Ante, p. 395.

2. In consequence of the equality of velocities at $r_{\prime}$ and $h_{\odot}$, t (the time of revolution) $\propto h$. If the mass ($\oplus \div$ ☽)$=81$, $(\frac{82}{81})^{\frac{3}{2}} \times 561.043 = 571.436$. In other words, *the actual* : *the experimental* value of h : : the virtual time of revolution of the Earth's centre, relatively to the Moon : the virtual time of revolution of the centre of gravity of the Earth and Moon. If this proportionality is exact, the value of $r_{\prime}$ is $\frac{tr}{\pi}\sqrt{\frac{g}{2h}} = 91{,}798{,}500$ miles.

3. The greatest distance of the Moon from the Earth is about $63r$, and $\frac{h}{h+r}$ is nearly equivalent to $\sqrt{\frac{1}{63}}$ If this equation is exact $h=571.25$; $r_{\prime}=91{,}813{,}400$.

4. The velocity acquired by falling through h, from the distance $r+h$, is nearly a mean proportional between the velocities of terrestrial rotation and revolution. If $h=569.363$, $r_{\prime}=91{,}965{,}500$ and the hourly velocity of revolution is $2 \times 3.14135 \times 91{,}965{,}500 \div 8766.153 = 65{,}911.7$; $3600 \times \sqrt{2gh} \div \frac{h+r}{r} = 8{,}280.6$, which is a mean proportional between 65,911.7 and 1,040.3. I can find no indication, in any of the planets or satellites, of a greater rotation-velocity than is thus indicated, and as it is difficult to conceive the possibility of such a velocity, I am inclined to regard this as the upper limit of possible value for $r_{\prime}$, and to believe, therefore, that the Sun's mean distance cannot be greater than 91,965,500 miles.

5. If we take h' a third proportional to r and h $\left(h'=\frac{h^2}{r}\right)$, the *vis viva* of rotation of an elastic particle at the Earth's surface : the *vis viva* at h' : : the force of gravity at h' : the force of gravity at the Earth's surface : : the velocity of light : the velocity which would be communicated by superficial gravity in one sidereal year.

$$497.827gt \div \left(\frac{r+h'}{r}\right)^2 = r_{\prime} = \frac{tr}{\pi}\sqrt{\frac{g}{2h}};$$

$$\therefore 497.827\pi \times r\sqrt{2g} = \frac{(r+h')^2}{\sqrt{h}}.$$

$$\text{If } h=573.967, \frac{r}{h}=6.90425;\ h'=83.1323;\ \frac{(r+h')^2}{\sqrt{h}}=683{,}279.4;$$

$$r_{\prime}=91{,}595{,}960.$$

We thus obtain five independent determinations, each of which is based upon considerations, some of which are necessary resultants of known mechanical laws, while others are expressive of actual circumstances of equilibrium, the greatest difference between any two of the results being less than one-half of one per cent. The experimental determination from the combustion of hydrogen (1), which differs less than one-hundredth of one per cent. from Hansen's estimate, and only one twenty-fifth of one per cent. from the mean of all the estimates, may, perhaps, be reasonably regarded as entitled to the greatest weight. The narrow compass within which they are all embraced, and the close approximation to the mean of the best astronomical computations, may be seen in the following table:

Newcomb (mean of two estimates)	92,266,000
TERRESTRIAL ROTATION (upper limit)	**91,965,500**
LUNAR DISTANCE	**91,813,400**
LUNAR MONTH	**91,798,500**
MEAN OF MECHANICAL ESTIMATES	**91,767,736**
Stone (mean of two estimates)	91,728,500
Hansen	91,672,000
HYDROGEN EXPLOSION (1)	**91,665,320**
Mean of Astronomical Estimates	91,636,300
VELOCITY OF LIGHT	**91,595,960**
Leverrier	91,329,000
Winnecke	91,186,000

Is it possible that there can be anything deceptive in these figures,—that any bias of unsuspected prejudice may have blinded me, or that I have been misled by mere fortuitous resemblances? The question whether there is not some conception of force which will unify centrifugal and centripetal, luminous, thermal, and gravitating action, is continually recurring; the accordance of Faraday's "lines of force" with lines of perfect fluidity, and the fundamental equation of oscillation, favor an affirmative answer; the increasing popularity of the theory that matter is nothing but force, prepares the way for every conceivable approximation and identification of molar and molecular laws.

If the undulations of light have any influence upon the gravity of bodies, the velocity of light being nearly uniform, its influence should tend to communicate a velocity as nearly like its own as gravity and inertia will allow, a tendency which is presumably most manifest in the most tenuous forms of matter. The gravitating velocity,

$\frac{r}{r+h}\sqrt{2gh} \propto \frac{1}{r+h}\sqrt{2mh}$, being a maximum when $r_{\odot} = h$ and $h \propto m$, it does not seem unreasonable to look for analogies between the extreme excursions of planets, satellites and gases, or between times, velocities, and living forces, in the direction of that maximum.

GENERAL RELATION OF AURORAS TO RAINFALL.

By Pliny Earle Chase.

(*Read before the American Philosophical Society, April 5th*, 1872.)

In order to ascertain if the parallelism, which I have pointed out between the daily rainfall and the frequency of auroras, can be traced in the annual curves, I have constructed the following table of normals, from data furnished by Lovering's Catalogue of Auroras, and Loomis's Meteor-